

## Summary of Convergence Tests for Infinite Series

Test	Series	Converges if	Diverges if	Comment
Divergence	$\sum_{n=1}^{\infty} a_n$		$\lim_{n \rightarrow \infty} a_n \neq 0$	cannot be used to show convergence
Geometric Series	$\sum_{n=0}^{\infty} ar^n$	$ r  < 1$	$ r  \geq 1$	Sum: $S = \frac{a}{1-r}$
$p$ -Series	$\sum_{n=1}^{\infty} \frac{1}{n^p}$	$p > 1$	$p \leq 1$	
Integral	$\sum_{n=1}^{\infty} a_n$ $a_n = f(n) \geq 0$	$\int_1^{\infty} f(x) dx$ converges	$\int_1^{\infty} f(x) dx$ diverges	$f$ is continuous, positive, and decreasing
Comparison	$\sum_{n=1}^{\infty} a_n$	$0 \leq a_n \leq b_n$ and $\sum_{n=1}^{\infty} b_n$ converges	$0 \leq b_n \leq a_n$ and $\sum_{n=1}^{\infty} b_n$ diverges	$a_n, b_n$ positive
Limit Comparison	$\sum_{n=1}^{\infty} a_n$	$\lim_{n \rightarrow \infty} \frac{a_n}{b_n} > 0$ and $\sum_{n=1}^{\infty} b_n$ converges	$\lim_{n \rightarrow \infty} \frac{a_n}{b_n} > 0$ and $\sum_{n=1}^{\infty} b_n$ diverges	$a_n, b_n$ positive Test fails if $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = 0$ or $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = \infty$
Alternating Series	$\sum_{n=1}^{\infty} (-1)^{n-1} b_n$	$b_{n+1} \leq b_n$ $\lim_{n \rightarrow \infty} b_n = 0$	$\lim_{n \rightarrow \infty} b_n \neq 0$	$b_n$ positive
Absolute Convergence	$\sum_{n=1}^{\infty} a_n$	$\sum_{n=1}^{\infty}  a_n $ converges		cannot be used to show divergence
Ratio	$\sum_{n=1}^{\infty} a_n$	$\lim_{n \rightarrow \infty} \left  \frac{a_{n+1}}{a_n} \right  < 1$	$\lim_{n \rightarrow \infty} \left  \frac{a_{n+1}}{a_n} \right  > 1$	Test fails if $\lim_{n \rightarrow \infty} \left  \frac{a_{n+1}}{a_n} \right  = 1$
Root	$\sum_{n=1}^{\infty} a_n$	$\lim_{n \rightarrow \infty}  a_n ^{1/n} < 1$	$\lim_{n \rightarrow \infty}  a_n ^{1/n} > 1$	Test fails if $\lim_{n \rightarrow \infty}  a_n ^{1/n} = 1$

**Comments:** The following general guidelines are useful:

1. Does the  $n$ th term approach zero as  $n$  approaches infinity? If not, the Divergence Test implies the series diverges.
2. Is the series one of the special types - geometric, telescoping,  $p$ -series, alternating series?
3. Can the integral test, ratio test, or root test be applied?
4. Can the series be compared favorably to one of the special types?